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STEADY STATE HEAT CONDUCTION MODELING BY THE GENERALIZED FINITE ELEMENT METHOD

Humberto Alves da Silveira Monteiro
Guilherme Garcia Botelho
Roque Luiz da Silva Pitangueira
Rodrigo Guerra Peixoto
Felício Bruzzi Barros
Department of Structural Engineering, Federal University of Minas Gerais. Av. Presidente Antônio Carlos, 6627, School of Engineering – Block 1, Pampulha, Belo Horizonte – MG, Brazil.
hmonteiro@ufmg.br; guilhermebotelho@ufmg.br; roque@dees.ufmg.br; rodrigo.peixoto@dees.ufmg.br; felicio@dees.ufmg.br

Abstract. *The Generalized Finite Element Method (GFEM) is a numerical technique suitable to solve a wide range of engineering continuum mechanics problems. Dating back to the mid-1990's, the GFEM is a relatively new numerical method which incorporates enrichment functions to the partition of unity and, by doing so, is more flexible and less mesh-dependent than the standard finite element formulation. In particular, it is a convenient tool in the study of heat transfer phenomena, being able to provide numerical solutions for the distribution of thermal energy inside a domain subjected to high temperature gradients. In that sense, this work presents the computational implementation of the GFEM to thermal problems. Validation examples of two-dimensional steady state conduction models are presented in order to illustrate the performance of the method in these cases. The work were executed in the INSANE system (INteractive Structural ANalysis Environment), a free software of high-level scientific research on numerical methods developed in the Department of Structural Engineering of the Federal University of Minas Gerais, Brazil.*

Keywords: *Steady State Heat Conduction, Heat Transfer, Generalized Finite Element Method.*

1. INTRODUCTION

In the study of continuum mechanics it is usual to adopt computational numerical methods to solve a variety of problems in order to obtain approximate solutions for some mathematical model. In general, the target problems are governed by intricate differential equations, with complex boundary conditions and bulky analytical solutions, which are hard (or impossible) to obtain most of the time. Thus numerical techniques emerge as an alternative to obtain quantitative (and qualitative) answers to unknown quantities within the scope of some study. General elasticity of solids, fluid mechanics as well as thermal analysis figure in the range of possibilities.

For instance, the heat transfer problem exists in various engineering situations. From the development of small electronic components, to the design of complex industrial structures, a fundamental element could be crucial: a close look to the temperature distribution (Jiji, 2009). This field variable can be defined as a scalar physical quantity that measures the average internal energy of a system in thermal equilibrium. That is, it can be considered as the description of the kinetic and potential energies of the particles within the system, related to their movements and due to their interactions with the external environment (and with each other), respectively (Borgnakke and Sontagg, 2008).

To mathematically model that phenomenon, a archetypal law provides the relationship between the heat flux and the temperature gradient within a domain. Based on experimental observations, the french physicist-mathematician Joseph Fourier, developed a equation, named after him, which states that the flux is directly proportional to the thermal conductivity of the material, for a given temperature gradient.

More specifically, the heat conduction science is primarily concerned with the study of elementary mechanisms in which the energy is transfered from particles of warmer regions of the domain to those lying in lower temperature areas, typical of solids (Özişik, 1993). It tries to fulfill two important questions: how does heat flux relate to temperature? And what governs the temperature distribution? (Jiji, 2009). In this regard, as Özişik (1993) states, although heat (or more precisely the heat flow) cannot be directly measured or observed, its nature has physical meaning an it is intrinsically related to the temperature gradient, which could vary over time (transient regime) or not (steady regime).

Although the Generalized Finite Element Method (GFEM) has been successfully used to model fracture mechanics problems (Belytschko and Black, 1999; Moës *et al.*, 1999; Duarte, 2001; Duarte and Kim, 2008; Kim *et al.*, 2009; Kim and Duarte, 2009, 2015) its use to study the thermo-mechanical behavior of solids still lacks of references in the literature. More recently, Hosseini *et al.* (2013) implemented a framework for fracture analysis of isotropic and orthotropic functionally graded materials under mechanical and steady state thermal loadings. Bencheikh *et al.* (2017) investigated the behavior of thin coating in plane heat transfer models using level-set enrichment functions. In the same way, Yu and Gong (2013) studied temperature distribution in heterogeneous media. Likewise, Zuo *et al.* (2015) examined cooling pipes in two-dimensional concrete domains and the evolution of temperature fields over time. O'Hara *et al.* (2009, 2011), in their turn, analyzed high gradients and transient regimes by global-local enrichments.

To contribute to the evolution of the area and develop a niche inside the research project (and department) in which this work lays down, the implementation of the GFEM to heat transfer problems was made. The INSANE (INteractive Structural ANalysis Environment) software was used as platform. This computational program is a high-level scientific research software developed in the Department of Structural Engineering of the Federal University of Minas Gerais, Brazil. It uses the object-oriented paradigm and its core is written in Java programming language.

In the following text we have first, in Section 2., the description of methodology aspects, with the formulation of the problem and numerical method, as well as some operational issues. In Section 3., the current stage of research is presented in parallel to some simple validation examples and study cases. At last, Section 4. shows final remarks an preliminary conclusions, followed by formal acknowledgments and references.

2. METHODOLOGY

First, the GFEM is presented. Next, the formulation of the target problem is developed. Finally, the implementation and the study cases are described in few words.

2.1 The Generalized Finite Element Method

2.1.1 Brief History

The Generalized Finite Element Method (GFEM) is a relatively new numerical method, developed in the mid-1990s. According to Duarte *et al.* (2000), its driving forces came from two independent sets of studies: (1) the works of Babuška *et al.* (1994) and Melenk (1995), in the form of the Special Finite Element Method and the Partition of Unit Method (PUM), respectively; and (2) the studies of Duarte (1995) and Duarte and Oden (1996) on meshless methods, more precisely the *H-p* Clouds Method.

It is worth mentioning that the GFEM enrichment strategy (section 2.1.2) is similar to the one of another numerical method, the Extended Finite Element (XFEM), initially developed in Belytschko and Black (1999) and Moës *et al.* (1999) for the study of crack propagation. According to Fries and Belytschko (2010), the distinction of nomenclature between PUM, GFEM and XFEM has become very confusing, and in practice, the methods may be qualified as identical.

2.1.2 Formulation

Initially, one may define the partition of unity (PU) as a set of functions whose sum equals the unity in all the points $\mathbf{x}$ of a domain Ω. The GFEM strategy consists of using the shape functions of standard finite elements (FEM) as PU, and another set of different linearly independent functions $\Im_j$ – local approximation functions (see Eq. 1) –, which multiplies the base N_j of each node $\mathbf{x}_j$ (Barros, 2002; Duarte *et al.*, 2000).

$$\Im_j = \{1,\, L_{j1}(\mathbf{x}),\, L_{j2}(\mathbf{x}),\, \ldots, L_{jq}(\mathbf{x})\} = \{L_{ji}(\mathbf{x})\}_{i=1}^{q} \tag{1}$$

Thus, the GFEM shape functions Φ_{ji} could be defined as follows:

$$\{\Phi_{ji}(\mathbf{x})\}_{i=1}^{q} = N_j(\mathbf{x}) \times \{L_{ji}(\mathbf{x})\}_{i=1}^{q} \tag{2}$$

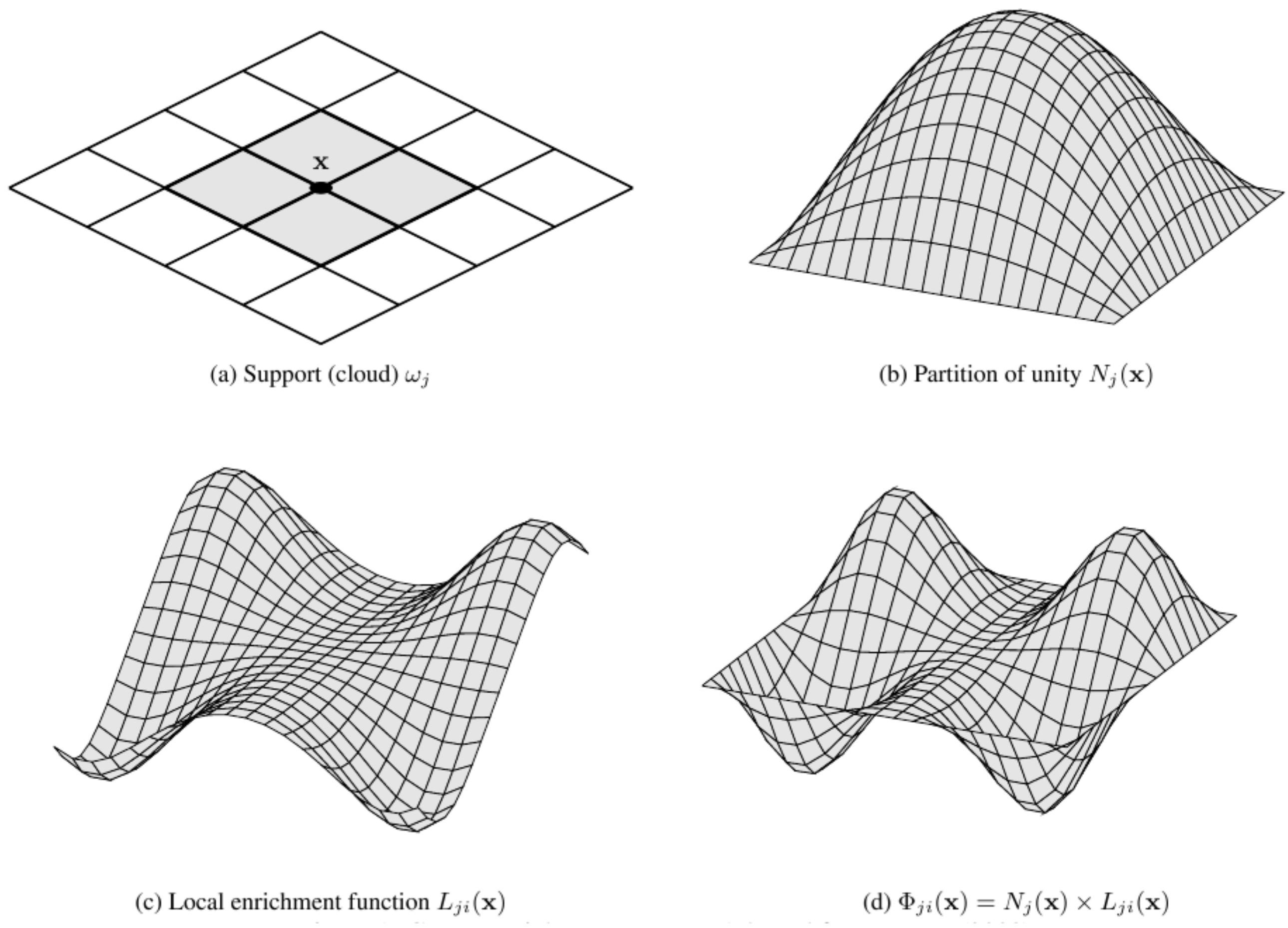
in which the local approximation functions $\{L_{ji}(\mathbf{x})\}_{i=1}^{q}$ could be polynomials or not, depending on the type of problem (Alves *et al.*, 2013).

Finally, the approximation of a generic scalar field variable is given by Eq. 3:

$$\theta(\mathbf{x}) = \sum_{j=1}^{n} N_j(\mathbf{x}) \left\{ \hat{\theta}_j + \sum_{i=1}^{q} L_{ji}(\mathbf{x}) \hat{\beta}_{ji} \right\} \tag{3}$$

where $\hat{\theta}_j$ and $\hat{\beta}_{ji}$ are nodal parameters associated with $N_j(\mathbf{x})$ and $N_j(\mathbf{x})L_{ji}(\mathbf{x})$, or in other words, to the FEM and GFEM approximations.

The figure 1 shows the enrichment function construction for a two-dimensional case. The PU is composed of a quadrilateral finite element mesh defined by a set of n nodes $\{\mathbf{x}_j\}_{i=1}^{n}$. In the figure, it is possible to identify the support ω_j, as well as the local and global approximation functions.



(a) Support (cloud) ω_j

(b) Partition of unity $N_j(\mathbf{x})$

(c) Local enrichment function $L_{ji}(\mathbf{x})$

(d) $\Phi_{ji}(\mathbf{x}) = N_j(\mathbf{x}) \times L_{ji}(\mathbf{x})$

Figure 1: GFEM enrichment strategy. Adapted from Barros (2002).

2.2 The Heat Transfer Problem

2.2.1 Mathematical Model Formulation

Consider a arbitrary control volume Ω, with a boundary Γ (Fig. 2). Let $\theta(\mathbf{x}, t)$ be a temperature field; $\mathbf{q}(\mathbf{x}, t)$, the heat flow (flux); $\rho(\mathbf{x}, t)$, the specific mass; $k(\mathbf{x}, t)$, the thermal conductivity; $c(\mathbf{x}, t)$, the specific heat.

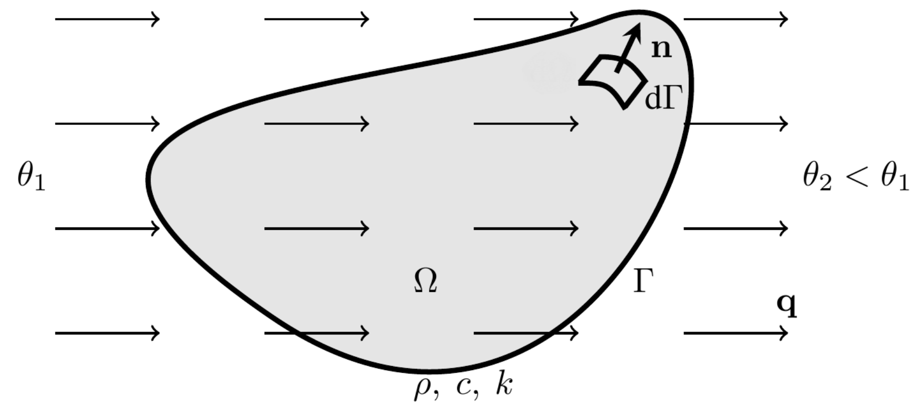


Figure 2: Control volume.

The Fourier law states (Eq. 4):

$$\mathbf{q} = -k\nabla\theta \tag{4}$$

in which the heat flux coming out the control volume can be defined by Eq. 5:

$$\oint_{\Gamma} \mathbf{q} \cdot \mathbf{n} d\Gamma = \int_{\Omega} (\nabla \cdot \mathbf{q}) d\Omega = \int_{\Omega} [\nabla \cdot (-k\nabla\theta)] d\Omega \tag{5}$$

Similarly, the work within the domain (e.g., chemical reactions) and the variation of internal energy could be defined, respectively, as:

$$\int_{\Omega} b(\mathbf{x}, t) d\Omega \quad \text{and} \quad \int_{\Omega} \rho c \frac{\partial \theta}{\partial t} d\Omega \tag{6}$$

Applying the conservation of energy, the Eq. 7 can be established:

$$\int_{\Omega} \left\{ \rho c \frac{\partial \theta}{\partial t} + [\nabla \cdot (-k\nabla\theta)] - b \right\} d\Omega = 0 \tag{7}$$

Next, assuming that the medium is homogeneous (ρ, c and k constants), it follows (Eq. 8):

$$\rho c \frac{\partial \theta(\mathbf{x}, t)}{\partial t} - k\nabla^2\theta(\mathbf{x}, t) - b(\mathbf{x}, t) = 0 \tag{8}$$

For steady state conduction, the Eq. 9 holds:

$$\frac{\partial \theta}{\partial t} = 0 \ ; \ \theta(\mathbf{x}, t) \to \theta(\mathbf{x}) \ ; \ b(\mathbf{x}, t) \to b(\mathbf{x}) \tag{9}$$

Finally, we have:

$$\nabla^2\theta(\mathbf{x}) = -\frac{1}{k} b(\mathbf{x}) \quad \text{and} \quad \nabla^2\theta(\mathbf{x}) = 0 \tag{10}$$

which are known as Poisson and Laplace equations (problems with and without internal heat generation), respectively.

2.2.2 Discrete Formulation

A (generalized) FEM-based approach for heat transfer problems starts by approximating the temperature field within a finite element by the interpolation of the temperature values at the nodes of this element. This approximation can be expressed mathematically as shown in Eq. 11:

$$\mathbf{\Theta} = \mathbf{\Phi}\hat{\mathbf{\Theta}} \ , \tag{11}$$

where $\mathbf{\Theta}$ is the temperature field within the finite element geometrical domain, $\hat{\mathbf{\Theta}}$ is a vector containing the degrees of freedom values (d.o.f.) at the finite element nodes and $\mathbf{\Phi}$ is a matrix containing pre-determined interpolating functions.

In the case of isotropic materials, the heat fluxes per unit area in each direction of the global coordinate system are determined by the Fourier law (Eq. 4) and can be expressed in matrix form by Eq. 12.

$$\mathbf{q}'' = -\mathbf{D}\mathbf{g} \tag{12}$$

in which $\mathbf{q}''$ are the heat fluxes per unit area in each direction of the global coordinate system, $\mathbf{D}$ is the constitutive matrix, shown illustratively in Eq. 13 for an isotropic material, and $\mathbf{g}$ is a vector containing the temperature gradient in each direction of the global coordinate system.

$$\mathbf{D} = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \tag{13}$$

where k is the thermal conductivity.

The temperature gradient vector mentioned in Eq. 12 can be expressed, for a cartesian system, by the matrix equation:

$$\mathbf{g} = \left\{ \frac{\partial \mathbf{\Theta}}{\partial x} \, , \, \frac{\partial \mathbf{\Theta}}{\partial y} \, , \, \frac{\partial \mathbf{\Theta}}{\partial z} \right\}^T \tag{14}$$

Writing the derivative operator, ∇, in matrix form,

$$\nabla = \left[\frac{\partial}{\partial x} \, , \, \frac{\partial}{\partial y} \, , \, \frac{\partial}{\partial z} \right]^T \tag{15}$$

and substituting Eq. 11 into Eq. 14 we get:

$$\mathbf{g} = (\nabla \mathbf{\Phi}) \hat{\mathbf{\Theta}} = \mathbf{B} \hat{\mathbf{\Theta}} \tag{16}$$

where $\mathbf{B}$ is the shape function derivatives operator calculated as the matrix product between ∇ and $\mathbf{\Phi}$.

The relationship between the actions on a finite element and the nodal temperatures is given by Eq. 17:

$$\mathbf{c} \hat{\mathbf{\Theta}} = \mathbf{f} \tag{17}$$

where $\mathbf{c}$ is the element conductivity matrix and $\mathbf{f}$ is a vector arising from the potentials to heat transfer acting in the finite element volume and on its surfaces. The vector $\mathbf{f}$ is called an equivalent nodal fluxes vector and has the three components:

$$\mathbf{f} = \mathbf{f_Q} + \mathbf{f_q} + \mathbf{f_h} \tag{18}$$

on which $\mathbf{f_Q}$ is the potential to heat transfer due to heat sources or heat sinks into the finite element volume, $\mathbf{f_q}$ is the potential to heat transfer due to a flux per unit area prescribed on any surface of the finite element, and $\mathbf{f_h}$ is the potential due to the heat transfer by convective surfaces.

The element conductivity matrix can be calculated as described in Eq. 19.

$$\mathbf{c} = \int_V \mathbf{B}^T \mathbf{D} \mathbf{B} dV + \int_{S_h} h \mathbf{\Phi}^T \mathbf{\Phi} dS \tag{19}$$

where h is the convective heat transfer coefficient.

The equivalent nodal fluxes vectors are calculated by Eq. 20, Eq. 21 and Eq. 22 :

$$\mathbf{f_Q} = \int_V \mathbf{\Phi}^T Q dV \tag{20}$$

where Q is the heat sink or heat source per unit volume;

$$\mathbf{f_q} = \int_S \mathbf{\Phi}^T q^* dS \tag{21}$$

where q^* is the heat flux per unit area prescribed on a surface of the element;

$$\mathbf{f_h} = \int_{S_h} \mathbf{\Phi}^T h U_{\mathrm{inf}} dS \tag{22}$$

where U_{inf} is the fluid temperature.

The contributions of each finite element can be properly assembled to obtain a algebraic system of equations representing the global behaviour of the body. The assembling process is based on the principles of equilibrium and continuity and is described in details by Logan (2007). The resulting set of equations can be written as:

$$\mathbf{C} \mathbf{\Theta} = \mathbf{F} \tag{23}$$

in which $\mathbf{C}$ is the global conductivity matrix of the body, $\mathbf{\Theta}$ represents the temperature field throughout the mesh and $\mathbf{F}$ is the vector containing the prescribed fluxes prescribed of the finite element mesh. This later vector ($\mathbf{F}$) contains heat fluxes from the element equivalent nodal fluxes vectors and from concentrated heat fluxes prescribed directly onto the nodes of the finite element mesh.

2.3 Implementation Aspects

As previously said, the INSANE is a free software, written in Java and by that uses the object-oriented programming (OOP). Besides standard and generalized FEM, the INSANE has formulations of Meshless Methods and Boundary Elements, as well as a vast library of constitutive models (Gori *et al.*, 2017), graphical tools and solver algorithms.

The INSANE numerical core have four principal instances (see Fig. 3): *Model*, *Assembler*, *Solution* and *Persistence*. The first one, represents the discrete model, the second one is responsible for handling and building the linear algebra components (described in the Section 2.2.2), the third one manages the solution process (solution of Eq. 23) and the last one takes care of the input and output mechanisms.

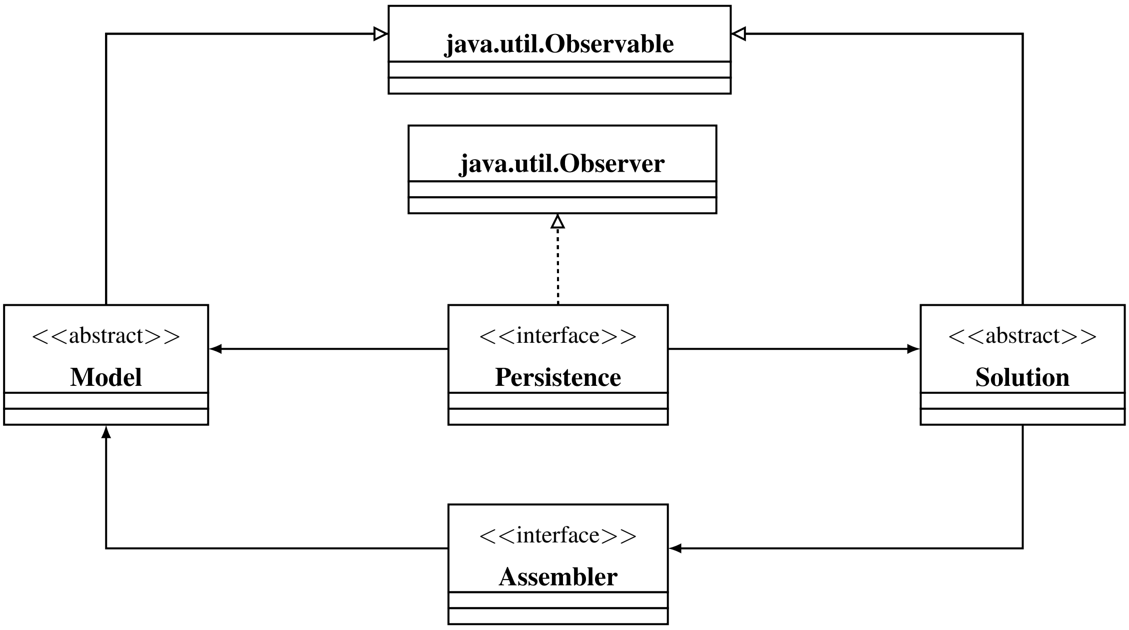


Figure 3: Simplified INSANE core.

The interventions made in this work are fundamentally related to the *Model* instance (but not restricted to it), in which specific classes were created to compute the integrals and derivatives of the discrete problem and to construct enriched shape functions.

2.4 Numerical Experiments

In order to validate the numerical model implemented here, simple one and two-dimensional heat conduction models, with well-known analytical solutions, were chosen. The numerical results will be compared to exact solutions in the next sections, in terms of spatial temperature distributions. The GFEM models were solved using coarse meshes and a single level of polynomial enrichment. As the motivation here is primarily academical and of ratification of the implementation, no special attention was paid to the material properties or discretization. For simplicity's sake, a uniform mesh was considered with a minimum number of nodes along the direction in which the analytical solution would be calculated.

3. RESULTS

In this section, a brief overview of the implemented code is presented in a simplified UML (Unified Modeling Language) manner. Also, the validation of the numerical method implementation is reported in relation to the analytical solution.

3.1 Brief Overview of the Implementation

Here we introduce a short description of the main classes built or modified for now: the problem driver, the analysis model and the enriched shape. In the next figures, the highlighted classes represent the ones implemented within the scope of this work. The UML diagrams only have illustration purposes; they just show some of the classes of each INSANE component and contextualize, from a software engineering point of view, the current implementation.

The *ProblemDriver* package is a set of classes (see Fig. 4) responsible for calculating, at element level, the matrices and vectors related to the mathematical problem being solved (Fonseca, 2008). It handles the integration process. In the field of heat transfer, these matrices are the elements conductivity and capacitive matrices; the vectors are the ones arising from the loadings applied in the element volume and over its surfaces (Botelho *et al.*, 2015).

The *AnalysisModel* classes (see Fig. 5) inform to the finite element and its integration points everything they need to know about the global analysis model, e.g., the number of degrees of freedom, the generalized strains and stresses, among other things (Fonseca, 2008).

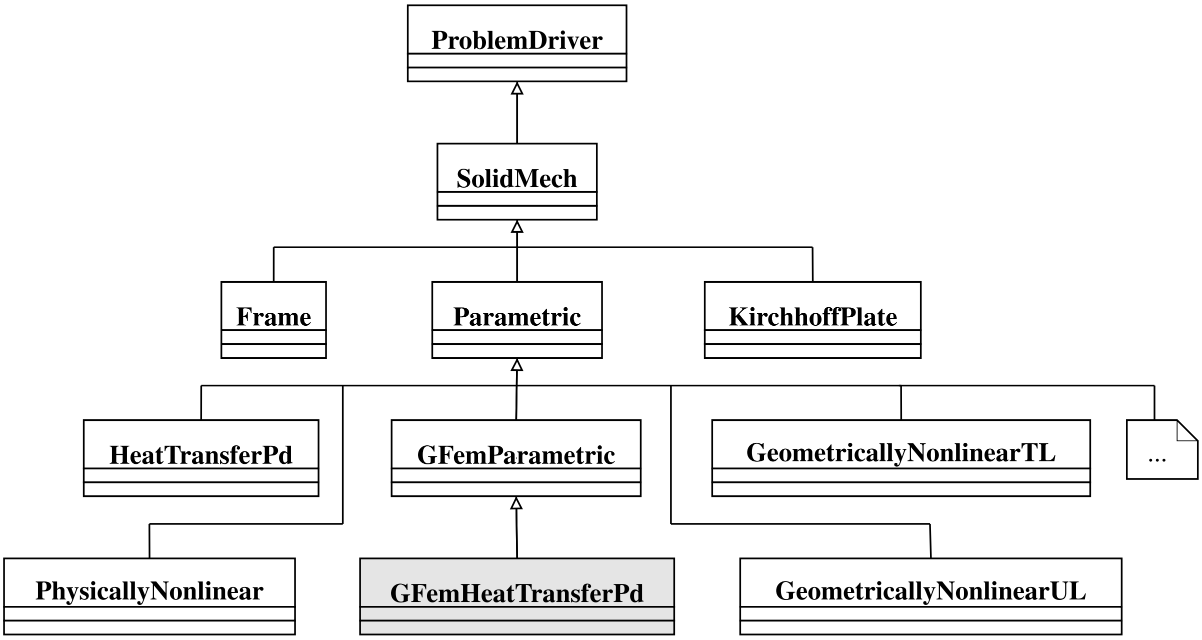


Figure 4: The *ProblemDriver* instances.

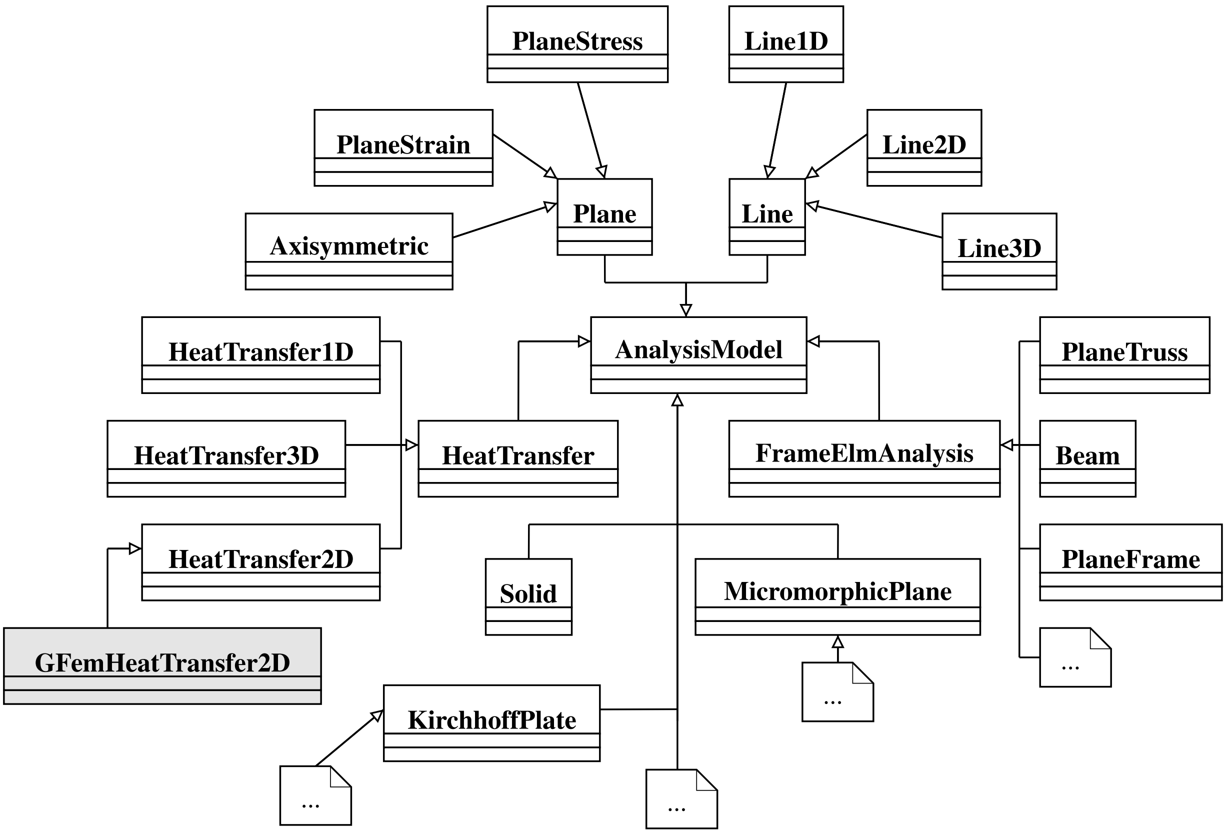


Figure 5: The *AnalysisModel* instances.

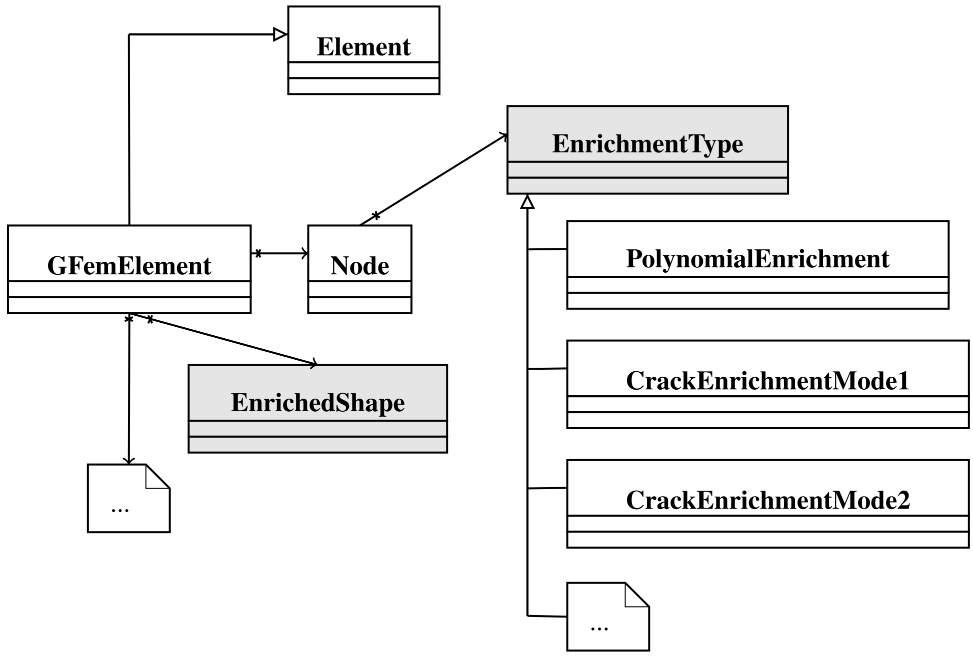


Figure 6: The *GFemElement* instance.

To handle the GFEM shape function and the operations of nodal enrichment, there are some special packages and classes. The generalized finite element instance has, among other attributes, an *EnrichedShape* reference which will compute the local approximation functions within the element. Each enriched element node can have different sets of enrichment functions, that could be polynomials, crack functions etc.

3.2 Validation Example: One-Dimensional Heat Conduction Model

The first validation example is a simple one-dimensional heat conduction problem. A plane wall insulated on the top and the bottom is submitted to different temperatures on its extremities. The domain has a width (L) of two length units. The material properties, including the conductivity (k), are unitary. The temperature on the left side (T_2) is 150 degrees Celsius and the temperature T_1 is equal to 50 degrees Celsius.

The analytical solution for this kind of problem (ignoring convective issues) can be stated, after Bergman *et al.* (2011), as follows:

$$\frac{d}{dx}\left(-k\frac{d\theta}{dx}\right) = 0 \tag{24}$$

After some algebraic manipulations, the following temperature field $\theta(x)$ can be written:

$$\theta(x) = (T_2 - T_1)\frac{x}{L} + T_1 \tag{25}$$

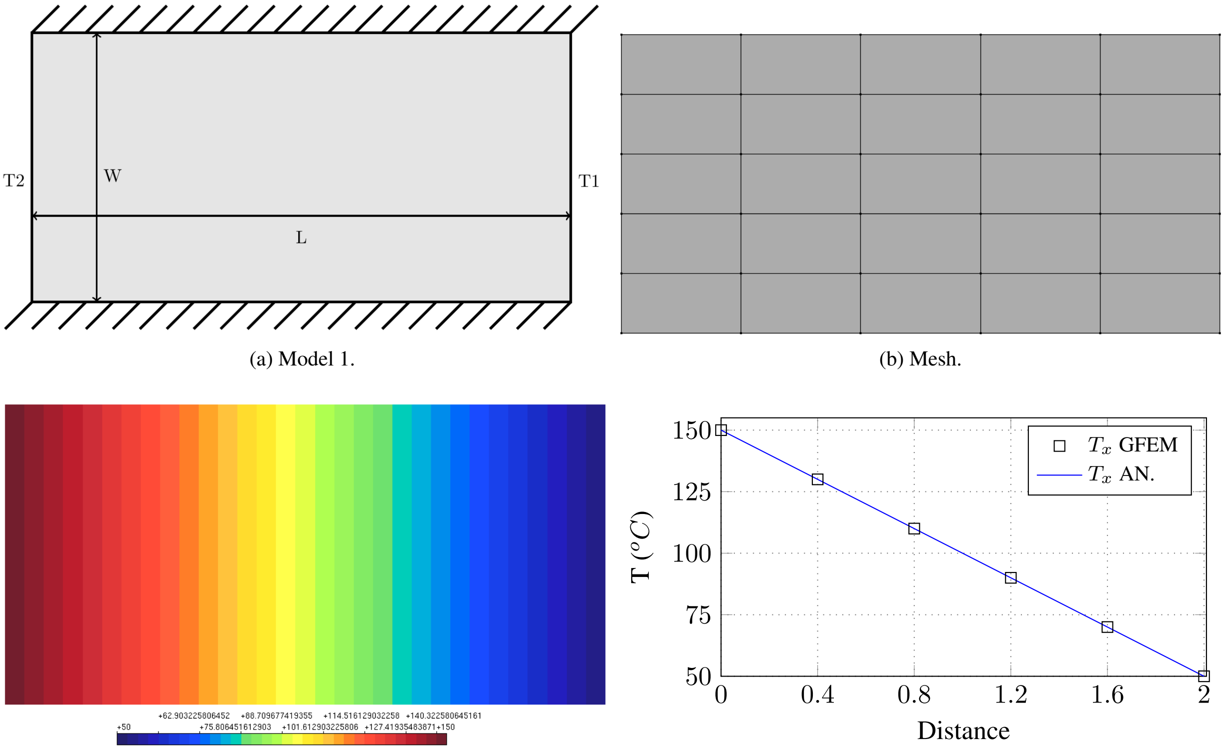
The problem is illustrated in Fig. 7a and was modeled using the GFEM mesh of Fig. 7b. The resulting temperature field is depicted in Fig. 7c and the comparison between the analytical solution of Eq. 25 and the numerical results is given by Fig. 7d.

3.3 Validation Example: Two-Dimensional Heat Conduction Model

The second validation example is a simple two-dimensional heat conduction problem. A plate is submitted to different temperatures on each of its sides. The domain has a width (L) of two length units and a height (W) of one length unit. The material properties, including the conductivity (k) are unitary. The temperature T_2 is 150 degrees Celsius and the temperature T_1 is equal to 50 degrees Celsius.

The analytical solution for this kind of problem can be stated, after Bergman *et al.* (2011), as follows:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \quad ; \quad \psi := \frac{\theta - T_1}{T_2 - T_1} \tag{26}$$



(a) Model 1.

(b) Mesh.

(c) Temperature field (^oC). $T(0.8, y) = 110$.

(d) Analytical (T_x AN.) and numerical solutions (T_x GFEM) in $y = 0.4$

Figure 7: Validation example 1: one-dimensional heat conduction.

After some algebraic manipulations, the following dimensionless temperature field $\psi(x, y)$ can be written:

$$\psi(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin(n\pi x/L) \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)} \tag{27}$$

The temperature field $\theta(x, y)$ can be obtained by Eq. 28:

$$\theta(x, y) = \psi \times (T_2 - T_1) + T_1 \tag{28}$$

For the proposed example (Fig. 8a), the temperature distribution was computed with the coarse GFEM mesh of Fig. 8b. Sixteen nodes along the region with bigger gradients were enriched with linear polynomials. The numerical results were compared with the analytical solution, expanded till its fifth non-zero term.

Figure 8c shows the temperature field and Fig. 8d depicts the comparison between the two solutions along two lines: one along the x-axis $[T_x \rightarrow (x \leq 1, y = 0.4)]$ and the other along the y-axis $[T_y \rightarrow (x = 0.8, y \leq 1)]$. As it could be seen, the GFEM results are in agreement with the analytical solution.

3.4 Study Case: Heat Transfer in a Flange of a Industrial Pipe Transporting High Temperature Fluid

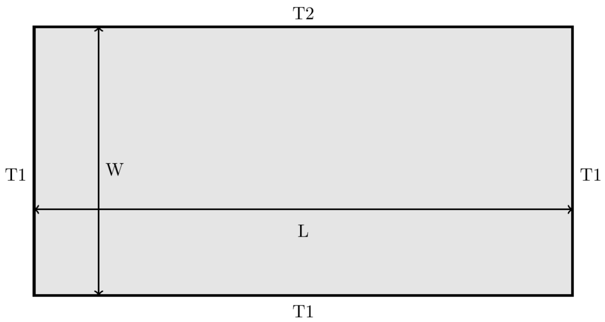
Here, a practical engineering problem is studied. In pipe systems design it is usual to face the problem of determining the temperature distribution throughout the section of a hot fluid transportation pipe, or even the thermal analysis of its forgings and auxiliary parts.

Moreover, in multiphysics problems, such as the thermo-mechanical one, it is imperative the heat transfer analysis as the temperature distribution affects the structural problem. By that, one needs to have the proper tools to handle the question, and when it comes to the development of a specific numerical method (like the GFEM), it is necessary to explore all the different physical problems.

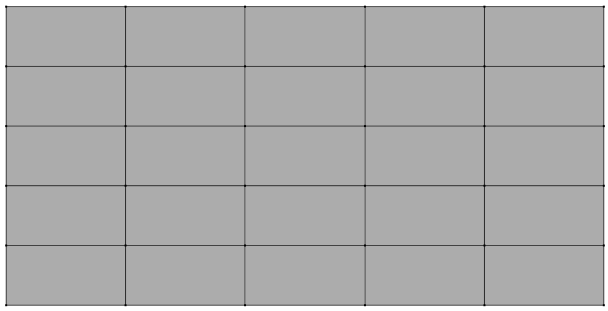
In this way, a simple steel carbon $1\frac{1}{2}$ flange of Material Group 1.1 and Class 300 (ASME B.16-5, 2009) were modeled. We present the temperature field and the comparison between the numerical results and a analytical solution of a disc.

For this kind of problem the analytical solution (in polar coordinates) can be stated as (Bergman *et al.*, 2011):

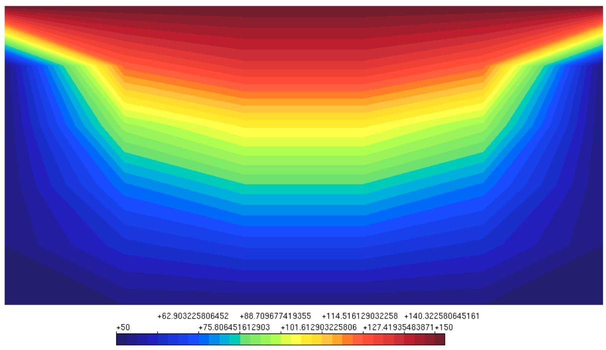
$$\frac{1}{r}\frac{d}{dr}\left(-kr\frac{d\theta}{dr}\right) = 0 \tag{29}$$



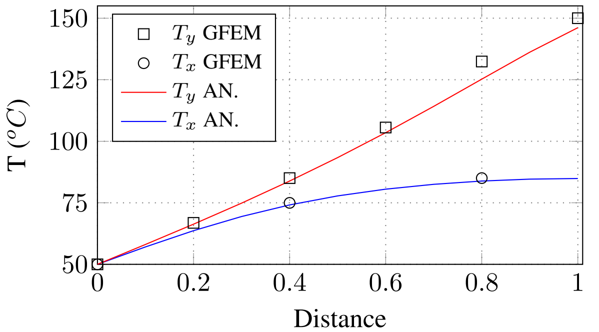
(a) Domain.



(b) Mesh.



(c) Temperature field (^oC). $T(0.8, 0.4) = 85.03$.



(d) Analytical (T_i AN.) and numerical solutions (T_i GFEM) in $(x, 0.4)$ and $(0.8, y)$.

Figure 8: Validation example 2: two-dimensional heat conduction.

Solving Eq. 29, the following radial temperature field can be obtained:

$$\theta(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_2 \tag{30}$$

in which T_1 and T_2 are the inner radius (r_1) and external radius (r_2) temperatures, respectively.

To solve this problem numerically, symmetry was taken in consideration and just one fourth of the flange was modeled. For simplicity, the holes for the bolts were not represented. A random triangular mesh (with three-noded elements) was generated and some internal nodes enriched with linear polynomials.

Figure 9 shows the schematics of the flange. Its dimensions are: $D = 155$mm; $Df = 114.3$mm; $df = \frac{7}{8}$in; $dt = NPS = 1\frac{1}{2}$in. The internal temperature is roughly taken as the maximum range possible for the material group (near 400^oC). The external temperature is 20^oC. Nonlinear effects and convective issues are completely ignored. Figure 10 shows the temperature field and the comparison to the analytical results of the radial temperature distribution along the x-axis.

4. CONCLUSION

Although the GFEM has proved its capacity to solve a wide range of physical problems, it still lacks of works on thermal analysis. With that in mind, this work shares the author's current attempts to model heat transfer by means of the Generalized Finite Element Method. The mathematical basis of the GFEM along with its formulation to basic heat conduction problems was registered. Some validation tests and results were given in parallel to analytical solutions. Applications to two-dimensional heat conduction were presented in a simple manner and the numerical results shown a good agreement with the analytical solutions.

The components registered here are just a first effort and take part of a bigger project currently been developed. At the present, multiphysics tools are being integrated to INSANE and a completely new framework (totally independent of discrete model – so the importance of applying GFEM to heat problems) is being implemented. Moreover, new enrichment functions which incorporates the behavior of a number of heat transfer problems are to be implemented. With that, it is expected that the GFEM will truly show its flexibility and capacity to represent really high and sharp temperature gradients, with less mesh dependency as possible.

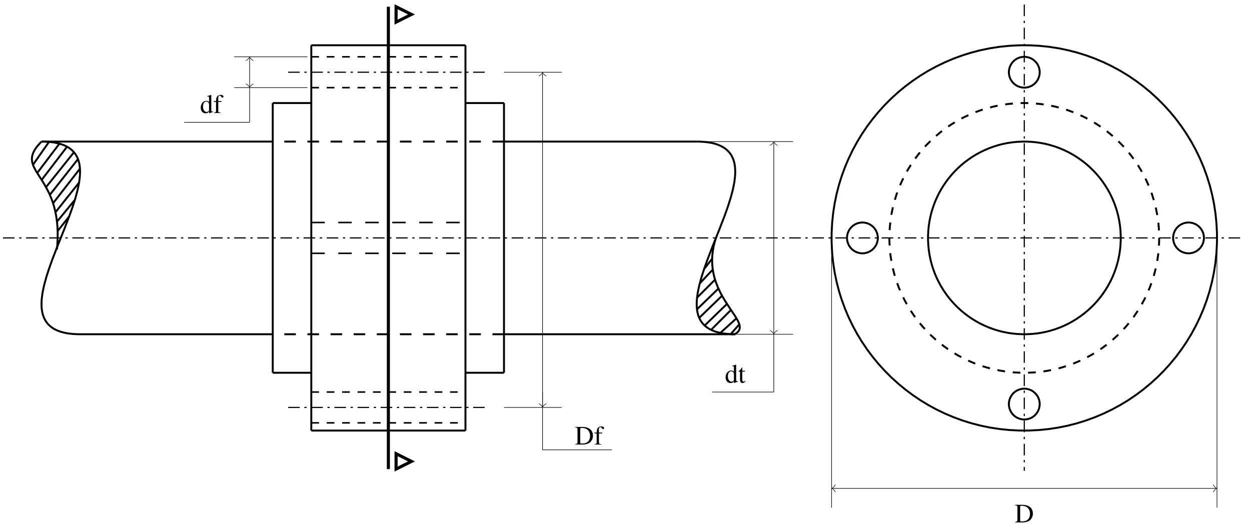
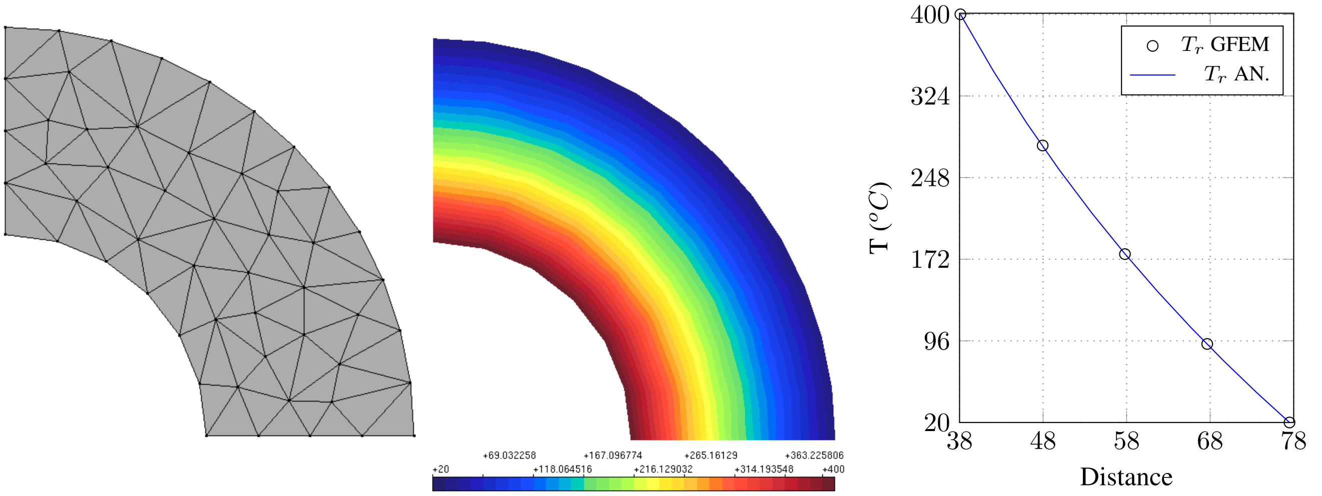


Figure 9: Flange schematics.



(a) Mesh.

(b) Temperature field (^oC).

(c) Analytical (T_r AN.) and numerical solutions (T_r GFEM) in ($x = 0.0$).

Figure 10: Study case: class 300 flange.

5. ACKNOWLEDGEMENTS

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